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POINTS:

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Chapters 12, 13, 14, parts of 15 (basic waves) in Giancoli.
November 25, 2015

NOTE: <u>NOTE: To get full credit, explain briefly what you are doing and which laws you are using in every step. Write legibly</u> USE DRAWINGS! Use the correct number of significant figures. Use vector symbols, arrows.

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1. [10] SOLUTIONS OF A COMPLEX NUMBER

[10] Find the four solutions of the complex equation $\hat{z}^4 = 3 + i4$. Calculate the second solution (n=1) in the form of a + ib.(4 significant figures)

$\hat{z}^4 = 3+4i = \sqrt{9+16}\; e^{i(\theta+2n\pi)}$ $\theta = 0.927$

$\hat{z}_K = 5^{\frac{1}{4}} e^{i\left(\frac{0.927}{4}+\frac{2K\pi}{4}\right)}$

$= 1.495\, e^{i(0.232+2K\cdot 0.785)}$ $K = 0,1,2,3$

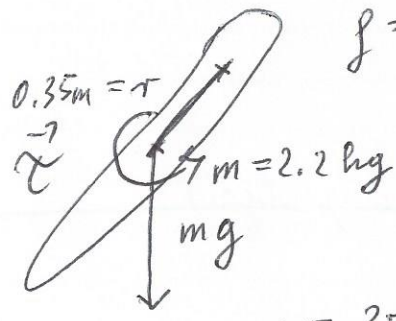
$\hat{z}_1 = 1.495\, e^{i(0.232+2\cdot 0.785)} = 1.495\, e^{i\,1.80}$

$= 1.495(\cos 1.80 + i \sin 1.80) = -0.344 + i\,1.46$

2. [10] A PHYSICAL PENDULUM

in the form of a planar object moves in simple harmonic motion with a frequency of 0.450Hz. The pendulum has a mass of 2.20 kg, and the pivot is located 0.350 m from the center of mass. Determine the moment of inertia about the pivot point and about the center of mass. Make a drawing and show the force on the pendulum and the torque arm.

$f = 0.45\,Hz$;



$\omega^2 \cancel{I_A} = \frac{mgr}{I_A} = \frac{2.2\cdot 9.8\cdot 0.35}{I_A}$

$\omega = 2\pi\cdot 0.45 = 2.827$; $\omega^2 = 7.994$

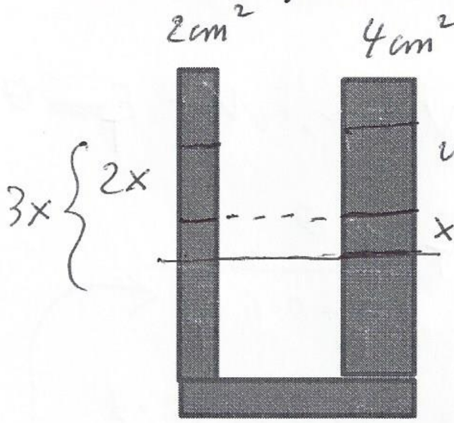
$I_A = 0.944\ kgm^2$

$I_{CM} = I_A - md^2 = I_A - 2.2\cdot 0.35^2$

$= 0.674\ kgm^2$

3. [10] FLUID STATICS

Mercury (ρ=13.6g/cm^3) is poured into a U-tube whose left arm has a cross-sectional area A_1 of 2.00 cm^2 and whose right arm has a cross-sectional area of A_2 of 4.00 cm^2. 100 grams of water are then poured into the right arm pushing the mercury in this arm down and the mercury in the left arm up. a) Determine the height of the water column in the right arm. b) By what distance does the mercury in the left arm rise? Explain which principles you are using.



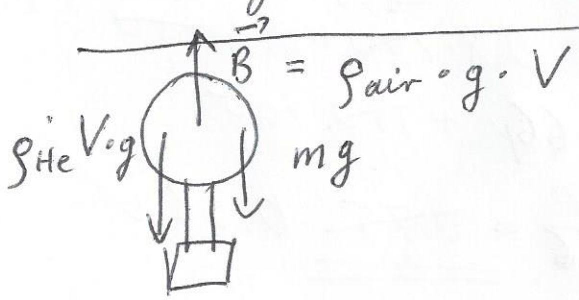
water : $\dfrac{100 \text{ cm}^3}{4 \text{ cm}^2} = 25$ cm column

pressure is same at this level $\rho g h$

left: $\rho_{Hg} \cdot \cancel{g} \cdot 3x = \rho_{H_2O} \cdot \cancel{g} \cdot 25$

$$x = \frac{1}{13{,}6} \cdot \frac{25}{3} = 0.6127 \text{ cm}$$

Hg rises by $2x = \underline{\underline{1{,}23 \text{ cm}}}$



4. [10] HELIUM BALLOON

[10] How many cubic meters of Helium are required to lift a balloon of 500kg payload to a height of 6000 m? (Take the density of Helium as 0.180kg/m^3) Assume the balloon maintains a constant volume and that the density of air decreases with the altitude according to $\rho_{air}(z) = \rho_0 e^{-\frac{z}{8000}}$, where z is the height in meters and ρ_0 is the density of air at sea level which is equal to 1.25kg/m^3. Make a drawing and show the exterior forces acting on the balloon.

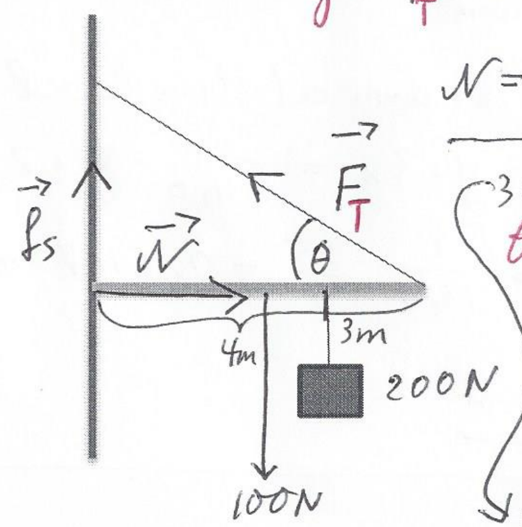
$$\rho_{air} \cdot g V = mg + \rho_{He} g V$$

$$(\rho_{air} - \rho_{He}) \cdot V = m \; ; \quad V = \frac{m}{\rho_{air} - \rho_{He}} = \frac{500}{0.41} = \underline{\underline{1218 \text{ m}^3}}$$

$$\rho_{air} = 1{,}25 \cdot e^{-\frac{6}{8}} = 0.590 \frac{\text{kg}}{\text{m}^3}$$

5. [10] STATIC EQUILIBRIUM

[10] One end of a 4.00 m long rod of weight F=100N is supported at its end by a cable attached to the vertical wall at an angle of θ=37.9 degrees between the horizontal rod and the cable. The other end of the rod rests against the wall, where it is held by friction. The coefficient of static friction is unknown. An additional weight of 200N hangs at the 3.00 m point, as measured from the wall along the rod. Find the coefficient of static friction which keeps the rod from slipping. Clearly show all the exterior forces and torques you use.



y: $F_T \cdot 0.6 + f_s = 300\,N = \mu_s \cdot N + F_T \sin\theta$

$N = F_T \cdot 0.8 = F_T \cdot \cos\theta$

$300 = \mu_s \cdot 0.8\, F_T + 0.6\, F_T$

torques:

$F_T \cdot 0.6 \cdot 4 = 100 \cdot 2 + 200 \cdot 3$

$2.4\, F_T = 800$

$F_T = \frac{800}{2.4} = \cancel{250}\ 333\ N$

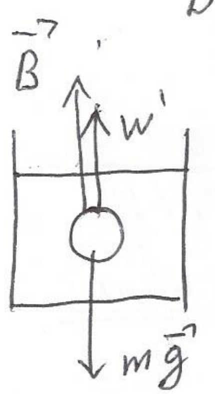
$300 = \mu_s \cdot 0.8 \cdot \cancel{250}\,333 + 0.6 \cdot \cancel{250}\,333$

$= 266\,\mu_s + \cancel{150}\ 200$

$\mu_s = \frac{100}{266} = \cancel{0.75}\ 0.38$ ✓

6. [10] DENSITY OF AN OBJECT

The weight of an object in air is 11.675 g and under water it is 8.543 g. Find the density of the object. The density of water is 1.000g/cm^3. Derive the equation for the density from Archimedes principle. Make a drawing and show the forces acting on the immersed mass.



$B = \rho g V_{liquid}$; $B + w' = mg = \rho V g$

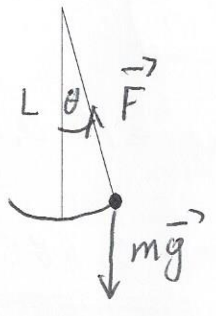
$\rho_L V g + w' = w = \rho V g \quad | : \rho V g$

$\frac{\rho_L}{\rho} + \frac{w'}{w} = 1$; $\frac{\rho_L}{\rho} = 1 - \frac{m'}{m}$

$\rho = \frac{\rho_L}{1 - \frac{m'}{m}} = \frac{1}{1 - \frac{8.543}{11.675}} = 3.73 \frac{g}{cm^3}$

7. [10] SIMPLE PENDULUM

A simple pendulum of length 1.68 m is released from an angle of 10.0 degrees with the vertical. **Derive** the differential equation for the pendulum from Newton's laws in vector form and find its period and angular frequency. (Project Newton's law on the tangential direction.) Indicate the angle, the forces and the acceleration or torque in the drawing.
Calculate the maximum speed and acceleration of the oscillating mass. Show the restoring force.



$\vec{F} + m\vec{g} = m\vec{a}$ $\quad \theta_0 = 10° = \frac{10}{180}\pi = 0.1745$

project onto $\vec{v}_\theta$

$-mg\sin\theta = m a_\theta$; $\sin\theta \to \theta$; $\omega_0 = 2.415\ s^{-1}$

$L\ddot{\theta} + g\theta = 0$; $\ddot{\theta} + \frac{g}{L}\theta = 0$; $\omega_0^2 = \frac{g}{L} = 5.833$

$\theta = \theta_0 \cos\omega_0 t$, $\dot{\theta} = -\omega_0\theta_0 \sin\omega_0 t$; $\dot{\theta}_{max} = \omega_0\theta_0 = 2.415 \cdot 0.1745$

$= 0.4215\ s^{-1}$

$a_{max} = \omega_0^2 \cdot \theta_0 \cdot L$ $\quad v_{max} = \dot{\theta}_{max} \cdot L = \underline{0.708\ \frac{m}{s}}$

$= 1.71\ \frac{m}{s^2}$

8. [10] WAVE ON A STRING

[5] Write an expression y(x,t) of a sine wave traveling to the left with an amplitude of 8.00 cm. λ=80.0cm, f=3.00Hz, y(0,t)=0 at t=0. Calculate the angular frequency, the angular wave number, and the speed of propagation of the wave. Show the dimensions for all quantities.

$y(x,t) = 8\,cm\,\sin\left(\frac{2\pi}{\lambda}\cdot x + 2\pi f \cdot t\right)$; $\quad \omega = 2\pi f = 18.8\ s^{-1}$

$k = \frac{2\pi}{\lambda} = 7.85\ m^{-1}$

$\underline{= 8\,cm \cdot \sin(7.85x + 18.8t)}$

$\underline{v = \lambda \cdot f = 2.40\ \frac{m}{s}}$

[5] Find the expression for the transversal speed and acceleration of a small segment of the string. What is the maximum values?

$v = \frac{\partial y}{\partial t} = 0.08 \cdot \cancel{7}\, 18.8 \cdot \cos(7.85x + 18.8t)$

$= 1.50\ \frac{m}{s}\cos(7.85x + 18.8t)$

$a = \frac{\partial^2 y}{\partial t^2} = -1.5\ \frac{m}{s} \cdot \frac{18.8}{s}\sin(7.85x + 18.8t)$

$= -28.3\sin(7.85x + 18.8t)$

9. [10] ENERGY OF A SPRING

A 325 g object connected to a spring with a force constant of 30.0 N/m oscillates on a horizontal, frictionless surface with an amplitude of 10.0 cm.
Find a) [3] the total energy of the system and b) [3] the speed of the object when the position x is 7.00cm. Find c) [2] the kinetic energy and d) [2] the potential energy when the position is 3.00 cm.

a) $E = \frac{1}{2} k A^2 = \frac{1}{2} \cdot 30 \cdot 0.01 = \underline{\underline{0.150 J}} = \underline{\underline{150 mJ}}$.

b) $0.150 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = 15 x^2 + \frac{0.325}{2} v^2 = 15 \cdot 0.07^2 + 0.1625 v^2$

$v^2 = \frac{0.15 - 0.0735}{0.1625} = 0.4707 \quad \underline{\underline{v = 0.686 \frac{m}{s}}}$

c) $K = E - \frac{1}{2} k x^2 = 0.15 - 15 \cdot 0.03^2 = 0.1365 J = 136.5 mJ$

$U = E - K = 0.0135 J = \underline{\underline{13.5 mJ}}$.

10. ~~[10] STANDING WAVES ON A STRING~~

~~Two sine waves with the same wavelength and frequency travel on a string in opposite directions. Both ends of the string are clamped down. The length of the string is L.~~
a) [5] Use the superposition formula for waves to find the formula for the resultant standing waves.

$y_1 = A \sin(kx - \omega t)$ $\qquad \lambda = 6 \cdot 10^{-7} m$

$f \cdot \lambda = c = 5 \cdot 10^{14} Hz$

$y_2 = A \sin(kx - \omega t + \phi)$

$y_1 + y_2 = 2 A \cos \frac{\phi}{2} \sin(Kx - \omega t + \frac{\phi}{2})$

$\frac{\phi}{2} = n \cdot \pi$ maxima

Transverse waves travel on a string with a speed of 30 m/s if the string is under a tension of ~~200~~ 10 $\frac{N}{m}$. What tension is required for a speed of 70 $\frac{m}{s}$ on the same string. $= \frac{11 g}{m}$ $\quad \mu = 0.0111 \frac{kg}{m}$

$v^2 = \frac{F}{\mu}$; $\mu = \cancel{\frac{30^2}{10} = 90}$ $\quad \mu = \frac{F}{v^2} = \frac{10}{900}$; $F = v^2 \mu = 70^2 \cdot \frac{10}{900} = 54.4 N$

~~b) [5] Derive the formulas for the possible wavelengths and frequencies of these standing waves, using the condition imposed on the sine wave for x=L. Calculate numerical values for the first two wavelengths and frequencies for L=2.00 m, mass of the string 20.0 grams, tension in the string 80.0 N~~

Transverse waves travel on a string with a speed of $30.0 \frac{m}{s}$ if the string is under a tension of $10.0 \frac{N}{m}$. What tension is required for a speed of $70.0 \frac{m}{s}$ on the same string?

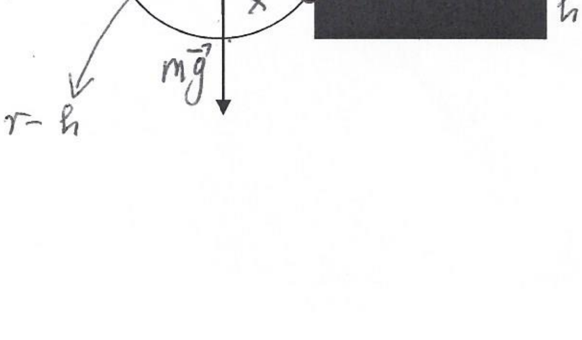
$$v^2 = \frac{F}{\mu} ; \quad \mu = \frac{F}{v^2} = \frac{10}{900} \frac{kg}{m} = 11.1\, g/m$$

$$F = v^2 \cdot \mu = 70^2 \cdot 0.0111 = \underline{\underline{54.4\ N}}$$

11. [10] STATIC EQUILIBRIUM, WHEEL

A uniform wheel of mass 20.0 kg and radius 0.500m must be rolled up a curb of height 20.0cm. Find the minimum tangential force necessary when applied horizontally at the top of the wheel. Label the forces correctly and show the torque arm vectors in the drawing. State the law which you are using.

$m = 20\ kg \quad r = 0.5\ m \quad h = 0.2\ m$



$F \cdot y = mg \cdot x ; \quad y = 2r - h = 0.8\ m$

$$x = \sqrt{r^2 - (r-h)^2} = \sqrt{0.25 - 0.3^2} = \sqrt{0.16} = 0.4$$

$$F \cdot 0.8 = 20 \cdot 9.8 \cdot 0.4$$

$$\underline{\underline{F = 98\ N}}$$

12. [10] SHOW THAT THE TIME RATE OF CHANGE of energy for an oscillator with a dampening factor b is given by $-bv^2$. To do so differentiate the total energy of a damped oscillator and use the differential equation of the oscillation.

$$ma + bv + kx = 0$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{dE}{dt} = mv\frac{dv}{dt} + kx\frac{dx}{x} = mva + kxv$$

$$= v\underbrace{(ma + kx)}_{-bv}$$

$$= -bv^2$$

Formulas:

$a) F_{damping} = -bv(t) \equiv -b\dot{x}$

$b) F = -kx - b\dot{x} = m\ddot{x}$ or

$c) \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0; \omega_0^2 = \frac{k}{m}; x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega_1 t + \varphi)$

$A = \dfrac{F/m}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + \left(\omega_f \frac{b}{m}\right)^2}}$ for forced oscillations

(1.1) simple pendulum: $\omega_0^2 = \frac{g}{l}$ physical pendulum $\omega_0^2 = \frac{mgr_A}{I_A}$

(1.2) $\hat{z} = Ae^{i\theta} = A(\cos\theta + i\sin\theta) = a + ib$; with $A = \sqrt{a^2 + b^2} = \sqrt{\hat{z}\hat{z}^*}$ $\theta = \arctan\frac{b}{a}$

(1.3) $\hat{z}^k = re^{i(\theta + 2n\pi)} = a + ib = \cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)$ has k solutions:

$\hat{z}_k = r^{\frac{1}{k}} e^{i\frac{(\theta + 2n\pi)}{k}}$ for n=0,1...k-1

$P = \frac{F}{A}$; the pressure in a liquid of depth y, measured from the surface is:

$$P = \rho g y; [P] = pascals = \frac{N}{m^2}; 1atm = 1.013 \times 10^5\, pa = 14.7 \frac{lbs}{in^2}$$

Pressure: $dF = PdA$

The buoyant force B equals the weight of the displaced liquid.

$$\frac{\rho}{\rho_{liquid}} = \frac{V_{liquid}}{V}; m = \rho V$$

Continuity equation;

$$A_1 \mathrm{v}_1 = A_2 \mathrm{v}_2$$

$$P_1 + \frac{1}{2}\rho \mathrm{v}_1^{\,2} + \rho g y_1 = P_2 + \frac{1}{2}\rho \mathrm{v}_2^{\,2} + \rho g y_2$$

Static equilibrium: $\begin{array}{l}\sum_i \vec{F}_i = \vec{0};\\ \sum_i \vec{\tau}_i = \vec{0}\end{array}$

$$a) A_1 \cos\theta_1 + A_1 \cos\theta_2 = 2A_1 \cos\frac{\theta_2 - \theta_1}{2} \cos\frac{\theta_2 + \theta_1}{2}$$

$$b) A_1 \sin\theta_1 + A_1 \sin\theta_2 = 2A_1 \cos\frac{\theta_2 - \theta_1}{2} \sin\frac{\theta_2 + \theta_1}{2}$$